# PHYS 798C Spring 2024
# Lecture 19 Summary

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## I. VORTEX ENERGY

We wish to first calculate the energy of a single vortex and then the interaction energy of two vortices. This will also allow us to calculate the force of interaction between two vortices, and finally the force exerted on a vortex by a transport current. All of this will be done in the extreme type-II limit $\kappa >> 1$ in which we effectively ignore the core of the vortex.

The energy per unit length of a vortex can be shown to be
$W'_{vortex} = \frac{1}{2\mu_0} \iint_{s_\perp} \vec{h}(\vec{r}) \cdot \vec{V}(\vec{r}) d^2 r$, where $\vec{h}$ is the magnetic field created by the vortices and $\vec{V}$ is their vorticity. Since the magnetic field of the single vortex came from solution of a linear equation, we shall assume that the vorticity and magnetic field can be formed from a linear superposition of single-vortex solutions in the case of multi-vortex problems.

For a single vortex, this evaluates to,
$W'_{vortex} = \frac{\Phi_0^2}{4\pi\mu_0\lambda_{eff}^2} K_0(\xi_{GL}/\lambda_{eff})$, where the Hankel function is evaluated at the edge of the core because the magnetic field is essentially the same at $r = 0$ as at $r = \xi_{GL}$. In the extreme type-II limit, the argument of $K_0$ is small, giving $K_0(x) \sim \ln(1/x)$ for $x << 1$. This yields,
$W'_{vortex} = \frac{\Phi_0^2}{4\pi\mu_0\lambda_{eff}^2} \ln(\kappa)$.
Note that the energy to create a vortex goes to zero as $T$ approaches $T_c$ (because $\lambda_{eff} \to \infty$), leading to a proliferation of vortex loops. This is one picture for how the superconductor to normal phase transition occurs in three dimensions. In two dimensions it is the first step in the Kosterlitz-Thouless phase transition.

## II. VORTEX INTERACTIONS

Two vortices a distance $\ell$ apart will have a total energy per unit length,
$W'_{2\ vortices} = 2\frac{\Phi_0^2}{4\pi\mu_0\lambda_{eff}^2} K_0(\xi_{GL}/\lambda_{eff}) \pm \frac{\Phi_0^2}{2\pi\mu_0\lambda_{eff}^2} K_0(\ell/\lambda_{eff})$. The first term is twice the self-energy of a vortex, while the second term is the interaction energy of the two vortices. The $\pm$ denotes the case of parallel $(+)$ and anti-parallel $(-)$ vortices. Like vortices repel, while opposites attract.

The interaction force can be deduced from the distance dependence of the interaction energy, $\vec{f}_{12} = -\partial W'_{2\ vortices}/\partial \ell = \pm \frac{\Phi_0^2}{2\pi\mu_0\lambda_{eff}^3} K_1(\ell/\lambda_{eff})$. This expression is proportional to the current created at vortex 2 by vortex 1: $\vec{J}_{12}$. It can be written as a Lorentz-like force as,
$\vec{f}_{12} = \vec{J}_{12} \times \Phi_0 \hat{z}$

If we imagine super-imposing two like vortices so that they share the same $z$ axis, this will create a doubling of the magnetic field and currents, as well as a doubling of the vorticity. Since the energy scales as $W'_{vortex} \propto \iint_{s_\perp} \vec{h}(\vec{r}) \cdot \vec{V}(\vec{r}) d^2 r$, it will increase by a factor of $2 \times 2 = 4$ compared to a single-flux-quantum vortex. This is much more energetically costly than just having two single-flux-quantum vortices a few $\lambda_{eff}$ away from each other. Hence 'giant vortices' are rarely encountered, and such vortices tend to dissolve into a collection of single-$\Phi_0$ vortices.

Note that if a vortex and anti-vortex super-impose, the superconducting electron density velocity field, as well as the magnetic fields, completely cancel out! This process is known as vortex-antivortex annihilation. The energy of the two vortices is then converted into quasiparticle excitations or phonons, and is essentially dissipated as 'heat'. It is possible to image trains of vortices and antivortices colliding and annihilating in a current-carrying superconducting strip, and several images, as well as a movie, are available on the class web site.